

## Boonton School District

<b>Course Title:</b>	EF Calculus	<b>Grade Level(s):</b>	11-12		
<b>Curriculum Area / Level:</b>	Math/ Honors	<b>Credits:</b>	5		
<b>Course prerequisites and/or co-requisites:</b>	Algebra I, Geometry, Algebra II				
<b>Course Description:</b>	This course covers all the topics needed to completely prepare the student for the Advanced Placement Calculus course or Honors Calculus course the following year. Students receive a strong foundation in the concept of functions including polynomial, trigonometric, logarithmic, and other special functions. Operations on functions, composition, limits, continuity, and inverses of functions are covered in depth. Differentiation and its applications are also covered.				
<b>Created by:</b>	Michael London	<b>Date:</b>	August, 2016	<b>BOE Approval:</b>	09/26/16
<b>District Equity Statement:</b>	As required by state law, it is the policy of Boonton School District not to discriminate on the basis of race, color, creed, religion, sex, ancestry, national origin, social or economic status, pregnancy, or physical handicap in its educational programs or activities and to maintain a learning environment that is free from sexual harassment. Courses of study and instructional materials shall be designed and selected in order to eliminate discrimination and promote understanding, sex equity, and mutual respect among people. No course offering, including but not limited to physical education, health, technology education, vocational, home economics, music and adult education, shall be limited on the basis of race, color, creed, religion, sex, ancestry, national origin, social or economic status, pregnancy, or physical handicap. Furthermore, there shall be no discrimination against students as to any educational activity or program because of pregnancy, childbirth, pregnancy-related disabilities, actual or potential parenthood, or family or marital status. If a student requests to be excluded or a physician certifies that such is necessary for her physical, mental, or emotional well-being, she must be provided with adequate and timely opportunity for instruction to continue or make up her schoolwork without prejudice or penalty.				

**Division of Umbrella & Mini Units**

**Umbrella Unit 1 Topic / Name:**

**Functions and their Graphs**

**Mini Unit(s) *(Add to the list of mini units as necessary)***

**1A. Algebraic Concepts**

**1B. Functions and their Graphs**

**1C. Polynomials and Rational Functions**

**Umbrella Unit 2 Topic / Name:**

**Exponential, Logarithmic, and Trigonometric Functions**

**Mini Unit(s) *(Add to the list of mini units as necessary)***

**2A. Exponential and Logarithmic Functions**

**2B. Trigonometry**

**2C. Trigonometric Graphs**

**Umbrella Unit 3 Topic / Name:**

**Trigonometric Equations and Identities**

**Mini Unit(s) *(Add to the list of mini units as necessary)***

**3A. Solving Trigonometric Equations**

**3B. Trigonometric Identities**

**Umbrella Unit 4 Topic / Name:**

**Trigonometric Applications, Analytic Geometry, and Systems of Equations**

**Mini Unit(s) *(Add to the list of mini units as necessary)***

**4A. Trigonometric Applications**

**4B. Analytic Geometry**

**4C. Systems of Equations**

## UMBRELLA UNIT 1

<b>Title:</b>	Functions and Their Graphs
<b>Duration:</b>	8 weeks
<b>Essential Questions:</b>	<ul style="list-style-type: none"><li>- What is the connection between linear functions and arithmetic sequences?</li><li>- What are the similarities and differences between solving a linear equation compared to a linear inequality?</li><li>- How is the domain and range of a function determined from the graph of the function?</li><li>- What happens to a parent function when you transform its graph?</li><li>- What is the algebraic process that is used to find the inverse of a function?</li><li>- What is the procedure that is used to find real zeros of a polynomial function?</li><li>- Why does every non-constant polynomial have a zero in the complex number system?</li><li>- How do you find the conjugate of a complex number?</li></ul>
<b>Summative Assessments:</b> (Assessment at the end of the learning period)	Quizzes Tests Graphing Project
<b>Formative Assessments:</b> (Ongoing assessments during the learning period)	<b>Homework:</b> assigned daily <b>Quizzes:</b> <ul style="list-style-type: none"><li>- Inverses</li><li>- Functions and One-to-One Functions</li><li>- Dividing Polynomials and the Remainder and Factor Theorems</li><li>- Graphs of Polynomial Functions</li><li>- Complex Numbers</li></ul> <b>Tests:</b> <ul style="list-style-type: none"><li>- Functions and Their Graphs</li><li>- Polynomial and Rational Functions</li></ul>

<b>Differentiation:</b>	Variations can be made to daily homework assignments for students achieving above or below the majority. Self-discovery labs can replace class discussions.
<b>CPI #</b>	<b>CUMULATIVE PROGRESS INDICATOR (CPI)</b>
<b>8.1.12.A.1</b>	<b>Construct a spreadsheet, enter data, and use mathematical or logical functions to manipulate data, generate charts and graphs, and interpret the results</b>
<b>21ST CENTURY LIFE AND CAREER (STANDARD 9)</b>	
<b>CPI #</b>	<b>CUMULATIVE PROGRESS INDICATOR (CPI)</b>
<b>9.1.12.A.1</b>	<b>Apply critical thinking and problem-solving strategies during structured learning experiences.</b>

<b>MINI UNIT 1A</b>	
<b>Title:</b>	<b>Algebraic Concepts</b>
<b>Duration:</b>	<b>2 weeks</b>
<b>Overview:</b>	<b>In this unit students will learn work with number patterns, sequences, and equations. The focus will be on functions and solving linear equations and inequalities. Students will recall information learned in Algebra II and expand on those concepts.</b>
<b>Essential Outcomes - Upon completion of this course students will know (declarative):</b>	<b>Alignment to Standards</b>
<b>How to identify number patterns and sequences</b>	<b>F.BF.1 F.IF.7</b>
<b>How to manipulate an equation</b>	<b>F.BF.1 F.IF.7</b>

Techniques to solve linear and quadratic equations and basic compound inequalities	A.CED.1 A.REI.3
<b>Essential Outcomes - Upon completion of this course students will be able to (procedural):</b>	<b>Alignment to Standards</b>
Work with number patterns, sequences, and equations	F.BF.1 F.IF.7
Solve linear and quadratic equations	A.CED.1 A.REI.3
Solve basic compound inequalities	A.CED.1 A.REI.3
<b>Essential Outcomes - Upon completion of this course students will understand (conceptual):</b>	<b>Alignment to Standards</b>
How to recognize patterns and sequences	F.BF.1 F.IF.7
How to identify correct methods of solving linear and quadratic equations	A.CED.1 A.REI.3
How to solve compound inequalities and the similarities/differences to equations	A.CED.1 A.REI.3
<b>Resources Mini Unit 1A:</b>	TI-84 Calculator, Textbook, Teaching Prompts, Interactive Whiteboard

<b>MINI UNIT 1B</b>	
<b>Title:</b>	<b>Functions and Their Graphs</b>
<b>Duration:</b>	<b>3 weeks</b>
<b>Overview:</b>	<b>In this unit students will learn to graph linear and quadratic functions. Students will be able to identify all parts of a parabolic graph including intercepts, vertex, and extremes. Operations and transformations of functions will be explored, including discovery of the inverse function.</b>
<b>Essential Outcomes - Upon completion of this course students will know (declarative):</b>	<b>Alignment to Standards</b>
<b>How to identify if a function is a relation</b>	<b>F.BF.1</b>
<b>The domain and range of a function</b>	<b>F.BF.1</b>
<b>How to identify the parts of a parabola based on the equation of the function</b>	<b>F.BF.1 F.IF.7</b>
<b>The difference between difference forms of the equation of a parabola</b>	<b>F.BF.1 F.IF.7</b>
<b>The manipulations of a parent function to create new functions</b>	<b>F.BF.1</b>
<b>Transformations of graphs</b>	<b>F.BF.1</b>
<b>The various properties of addition, subtraction, multiplication, and division of functions</b>	<b>F.BF.7</b>
<b>Properties of composite functions and their domains</b>	<b>F.BF.7</b>
<b>The difference between odd and even functions</b>	<b>F.BF.1 F.IF.7</b>
<b>Inverse relations and functions</b>	<b>F.BF.4</b>

<b>Average rates of change over an interval</b>	<b>F.BF.1</b>
<b>Essential Outcomes - Upon completion of this course students will be able to (procedural):</b>	<b>Alignment to Standards</b>
<b>Determine whether a function is a relation</b>	<b>F.BF.1</b>
<b>Find the domain of a function</b>	<b>F.BF.1</b>
<b>Determine a graph to be a function</b>	<b>F.BF.1 F.IF.7</b>
<b>Identify the parts of a parabola based on the equation of the function</b>	<b>F.BF.1 F.IF.7</b>
<b>Convert from one form of a quadratic function to another</b>	<b>F.BF.1 F.IF.7</b>
<b>Determine parent functions</b>	<b>F.BF.1</b>
<b>Transform graphs of parent functions</b>	<b>F.BF.1</b>
<b>Form sum, difference, product, and quotient functions and find their domains</b>	<b>F.BF.7</b>
<b>Form composite functions and find their domains</b>	<b>F.BF.7</b>
<b>Determine whether a function is even or odd</b>	<b>F.BF.1 F.IF.7</b>
<b>Define inverse relations and functions</b>	<b>F.BF.4</b>
<b>Find inverse relations from tables, graphs, and equations</b>	<b>F.BF.4</b>
<b>Determine whether an inverse relation is a function</b>	<b>F.BF.4</b>

Find the average rate of change of a function over an interval	F.BF.1
Work with and solve various problems involving average rate of change	F.BF.1
<b>Essential Outcomes - Upon completion of this course students will understand (conceptual):</b>	<b>Alignment to Standards</b>
When a function is and is not a relation	F.BF.1
The difference between a linear and parabolic function	F.BF.1
The difference between a linear and parabolic graph	F.BF.1 F.IF.7
The different parts of a parabola and how to identify them algebraically and graphically	F.BF.1 F.IF.7
The relationship between the different forms of a quadratic equation	F.BF.1 F.IF.7
Parent functions how to manipulate them into other functions and identify them a graph	F.BF.1 F.IF.7
Whether a function has an inverse	F.BF.4
How to find an inverse through various methods	F.BF.4
Average rate of change	F.BF.1
<b>Resources Mini Unit 1B:</b>	TI-84 Calculator, Textbook, Teaching Prompts, Interactive Whiteboard



<b>MINI UNIT 1C</b>	
<b>Title:</b>	<b>Polynomial and Rational Functions</b>
<b>Duration:</b>	<b>3 weeks</b>
<b>Overview:</b>	<b>This unit focuses on Polynomials. Students learn the properties of polynomials and how to graph them. Time is spent with the Zero Principal and factoring polynomials using a variety of methods.</b>
<b>Essential Outcomes - Upon completion of this course students will know (declarative):</b>	<b>Alignment to Standards</b>
<b>Properties of polynomials</b>	<b>A.APR.6 A.APR.7</b>
<b>The Remainder and Factor Theorems of Polynomials</b>	<b>A.APR.6 A.APR.7</b>
<b>Zero Principle of polynomials</b>	<b>A.APR.6 A.APR.7</b>
<b>Various methods of factoring a polynomial</b>	<b>A.APR.6 A.APR.7</b>
<b>Graphs and properties of various polynomial functions</b>	<b>A.APR.6 A.APR.7</b>
<b>A complete understanding of complex numbers</b>	<b>N.CN.3 N.CN.8</b>
<b>The Fundamental Theorem of Algebra</b>	<b>N.CN.9</b>
<b>Essential Outcomes - Upon completion of this course students will be able to (procedural):</b>	<b>Alignment to Standards</b>

Define and divide polynomials	A.APR.6 A.APR.7
Apply the Remainder and Factor Theorems and make connections between them	A.APR.6 A.APR.7
Determine the maximum number of zeros of a polynomial	A.APR.6 A.APR.7
Factor a polynomial completely	A.APR.6 A.APR.7
Recognize and describe the graphs of various polynomial functions	A.APR.6 A.APR.7
Find the conjugate of a complex number	A.APR.6 A.APR.7
Simplify square roots of negative numbers	A.APR.6 A.APR.7
Use and apply the Fundamental Theorem of Algebra	N.CN.9
<b>Essential Outcomes - Upon completion of this course students will understand (conceptual):</b>	<b>Alignment to Standards</b>
All properties of polynomials including manipulations, Remainder and Factor Theorems, maximum number of zeros, and rational zeros	A.APR.6 A.APR.7
When and how to use various methods of factoring	A.APR.6 A.APR.7
The properties of complex numbers	A.APR.6 A.APR.7

<b>The uses for the Fundamental Theorem of Algebra</b>	<b>N.CN.9</b>
<b>Resources Mini Unit 1C:</b>	TI-84 Calculator, Textbook, Teaching Prompts, Interactive Whiteboard

## UMBRELLA UNIT 2

<b>Title:</b>	Exponential, Logarithmic, and Trigonometric Functions
<b>Duration:</b>	9 weeks
<b>Essential Questions:</b>	<ul style="list-style-type: none"><li>- How are exponential and logarithmic models used along with polynomial models to solve real world situations?</li><li>- How is right triangle trigonometry used to solve right triangles?</li><li>- How is the unit circle used to describe trigonometric functions?</li><li>- Describe how do you convert between radians and degrees?</li><li>- What are the relationships between the Pythagorean Identities for Trigonometry?</li><li>- How do you graph the basic trigonometric functions in the coordinate plane?</li><li>- How do transformations affect the trigonometric graphs of each function?</li><li>- How do you use graphs of trigonometric functions to determine trigonometric identities?</li></ul>
<b>Summative Assessments:</b> (Assessment at the end the learning period)	Benchmark Post Test covering all concepts covered in Unit 2
<b>Formative Assessments:</b> (Ongoing assessments during the learning period)	<b>Homework:</b> assigned daily for new topics <b>Quizzes:</b> <ul style="list-style-type: none"><li>- Solving Exponential Equations</li><li>- Trigonometric Ratios</li><li>- Right Triangle Trigonometry</li></ul>

	<ul style="list-style-type: none"> <li>- Coterminal Angles</li> <li>- Radian Degree Measures</li> <li>- Unit Circles and Reference Angles</li> <li>- Quotient, Reciprocal, and Pythagorean Identities</li> <li>- Graphing Sine, Cosine, and Tangent Functions</li> </ul> <p><b>Tests:</b></p> <ul style="list-style-type: none"> <li>- Trigonometry</li> <li>- Trigonometric Graphs</li> </ul>
<b>Differentiation:</b>	Variations can be made to daily homework assignments for students achieving above or below the majority. Self-discovery labs can replace class discussions.
<b>CPI #</b>	<b>CUMULATIVE PROGRESS INDICATOR (CPI)</b>
<b>8.1.12.A.1</b>	<b>Construct a spreadsheet, enter data, and use mathematical or logical functions to manipulate data, generate charts and graphs, and interpret the results</b>
<b>21ST CENTURY LIFE AND CAREER (STANDARD 9)</b>	
<b>CPI #</b>	<b>CUMULATIVE PROGRESS INDICATOR (CPI)</b>
<b>9.1.12.A.1</b>	<b>Apply critical thinking and problem-solving strategies during structured learning experiences.</b>

<b>MINI UNIT 2A</b>	
<b>Title:</b>	<b>Exponential and Logarithmic Functions</b>
<b>Duration:</b>	<b>2 Weeks</b>
<b>Overview:</b>	<b>This unit focuses on exponential and logarithmic functions. Student will learn how to solve both types of equations through a variety of methods. Students will understand the relationship</b>

<b>between exponential and logarithmic functions.</b>	
<b>Essential Outcomes - Upon completion of this course students will know (declarative):</b>	<b>Alignment to Standards</b>
Several methods to solve exponential equations	<b>F.BF.5</b>
How to solve logarithmic equations	<b>F.BF.5</b>
Methods to solve a variety of application problems by using exponential and logarithmic equations	<b>F.BF.5</b>
<b>Essential Outcomes - Upon completion of this course students will be able to (procedural):</b>	<b>Alignment to Standards</b>
Solve exponential equations	<b>F.BF.5</b>
Solve logarithmic equations	<b>F.BF.5</b>
Solve a variety of application problems by using exponential and logarithmic equations	<b>F.BF.5</b>
<b>Essential Outcomes - Upon completion of this course students will understand (conceptual):</b>	<b>Alignment to Standards</b>
The difference between exponential and logarithmic equations	<b>F.BF.5</b>
When to apply exponential and logarithmic equations to solve a real-life problem	<b>F.BF.5</b>
<b>Resources Mini Unit 2A:</b>	TI-84 Calculator, Textbook, Teaching Prompts, Interactive Whiteboard

<b>MINI UNIT 2B</b>	
<b>Title:</b>	<b>Trigonometry</b>
<b>Duration:</b>	<b>4 weeks</b>
<b>Overview:</b>	<b>This unit focuses on trigonometry. Students will learn how to use the six trigonometric functions to solve right triangles. Students will learn how to apply these concepts to real-life problems. Students will learn the radian angle measures and how to convert from degrees to radians.</b>
<b>Essential Outcomes - Upon completion of this course students will know (declarative):</b>	<b>Alignment to Standards</b>
<b>The six trigonometric ratios of an acute angle in terms of a right triangle</b>	<b>F.TF.3</b>
<b>How to apply trigonometric ratios using a calculator</b>	<b>F.TF.3</b>
<b>Right triangles are used to solve real-life problems</b>	<b>F.TF.3</b>
<b>Negative angles and angles greater than 180 degrees</b>	<b>F.TF.3</b>
<b>Radian angle measures</b>	<b>F.TF.3</b>
<b>Trigonometric ratios in the coordinate plane</b>	<b>F.TF.3</b>
<b>Trigonometric ratios in the unit circle</b>	<b>F.TF.3</b>
<b>Tangent lines to a curve</b>	<b>F.TF.4</b>
<b>Essential Outcomes - Upon completion of this course students will be able to (procedural):</b>	<b>Alignment to Standards</b>

<b>Define the six trigonometric ratios of an acute angle in terms of a right triangle</b>	<b>F.TF.3</b>
<b>Use a calculator to apply trigonometric ratios to solve problems</b>	<b>F.TF.3</b>
<b>Apply right triangle concepts to solve real-life problems</b>	<b>F.TF.3</b>
<b>Use a rotating ray to extend the definition of angle measure to negative angles and angles greater than 180 degrees</b>	<b>F.TF.3</b>
<b>Define radian measure and convert angle measures between degrees and radians</b>	<b>F.TF.3</b>
<b>Define the trigonometric ratios in the coordinate plane</b>	<b>F.TF.3</b>
<b>Define trigonometric functions in terms of the unit circle</b>	<b>F.TF.3</b>
<b>Construct a tangent line from a point outside of the unit circle</b>	<b>F.TF.4</b>
<b>Essential Outcomes - Upon completion of this course students will understand (conceptual):</b>	<b>Alignment to Standards</b>
<b>When and which trigonometric ratios to solve problems</b>	<b>F.TF.3</b>
<b>When to apply trigonometric ratios to solve real-life problems</b>	<b>F.TF.3</b>
<b>The relationship between reference angle and an angle greater than 180 degrees and negative angles</b>	<b>F.TF.3</b>



The relationship between radians and degrees	F.TF.3
How trigonometric ratios are used in the coordinate plane	F.TF.3
Trigonometric ratios in the unit circle	F.TF.3
The concept of a tangent line of a curve	F.TF.4
<b>Resources Mini Unit 2B:</b>	TI-84 Calculator, Textbook, Teaching Prompts, Interactive Whiteboard

<b>MINI UNIT 2C</b>	
<b>Title:</b>	<b>Trigonometric Graphs</b>
<b>Duration:</b>	<b>3 weeks</b>
<b>Overview:</b>	<b>This unit focuses on the graph of trigonometric graphs. Students will learn how to graph the sine, cosine, and tangent functions. They will recognize the domain and range of these functions. Students will be able to transform trigonometric functions and be able to identify the period and amplitude. Given a graph, students will be able to identify the trigonometric function.</b>
<b>Essential Outcomes - Upon completion of this course students will know (declarative):</b>	<b>Alignment to Standards</b>
Trigonometric graphs	F.TF.4 F.TF.6
The domain and range of trigonometric functions	F.TF.4 F.TF.6
Transformations of trigonometric functions	F.TF.4 F.TF.6

<b>Odd and even functions</b>	<b>F.TF.4 F.TF6</b>
<b>Graphs of the tangent functions</b>	<b>F.TF.4 F.TF6</b>
<b>The period and amplitude of basic trigonometric functions</b>	<b>F.TF.4 F.TF6</b>
<b>To identify graphs of the trigonometric functions</b>	<b>F.TF.4 F.TF6</b>
<b>Essential Outcomes - Upon completion of this course students will be able to (procedural):</b>	<b>Alignment to Standards</b>
<b>Graph the basic trigonometric functions</b>	<b>F.TF.4 F.TF6</b>
<b>State the domain and range of these trigonometric functions</b>	<b>F.TF.4 F.TF6</b>
<b>Graph transformations of the basic trigonometric functions</b>	<b>F.TF.4 F.TF6</b>
<b>Use the unit circle to explain symmetry functions</b>	<b>F.TF.4 F.TF6</b>
<b>Identify the period and amplitude of trigonometric functions</b>	<b>F.TF.4 F.TF6</b>
<b>Essential Outcomes - Upon completion of this course students will understand (conceptual):</b>	<b>Alignment to Standards</b>
<b>The parts of the trigonometric functions</b>	<b>F.TF.4</b>

	F.TF6
How domain and range work with trigonometric functions	F.TF.4 F.TF6
Transformations of trigonometric functions	F.TF.4 F.TF6
Trigonometric functions in terms of the unit circle	F.TF.4 F.TF6
<b>Resources Mini Unit 2C:</b>	TI-84 Calculator, Textbook, Teaching Prompts, Interactive Whiteboard

### UMBRELLA UNIT 3

<b>Title:</b>	Trigonometric Equations, Identities, and Applications
<b>Duration:</b>	8 weeks
<b>Essential Questions:</b>	<ul style="list-style-type: none"><li>- How do you graphically solve a trigonometric equation?</li><li>- What is the difference between sine function and the restricted sine function and why is it important when working with the inverse sine function?</li><li>- How is proving or verifying a trigonometric identity different than solving a trigonometric equation?</li><li>- What is the difference between the reciprocal and co-functional relationships for trigonometric functions?</li><li>- How can the double-angle identity for sine be used to calculate a distance?</li></ul>
<b>Summative Assessments:</b> (Assessment at the end the learning period)	Benchmark Post Test covering all concepts covered in Unit 3
<b>Formative Assessments:</b> (Ongoing assessments during the learning period)	<p><b>Homework:</b> to be given daily on each introduced topic</p> <p><b>Quizzes:</b> Solving Trigonometric Equations Graphically Basic Trigonometric Identities Trigonometric Addition and Subtraction Properties Using Double Angle Identities</p> <p><b>Tests:</b> Solving Trigonometric Equations</p>

	Trigonometric Identities
<b>Differentiation:</b>	Variations can be made to daily homework assignments for students achieving above or below the majority. Self-discovery labs can replace class discussions.
<b>CPI #</b>	<b>CUMULATIVE PROGRESS INDICATOR (CPI)</b>
<b>8.1.12.A.1</b>	<b>Construct a spreadsheet, enter data, and use mathematical or logical functions to manipulate data, generate charts and graphs, and interpret the results</b>
<b>21ST CENTURY LIFE AND CAREER (STANDARD 9)</b>	
<b>CPI #</b>	<b>CUMULATIVE PROGRESS INDICATOR (CPI)</b>
<b>9.1.12.A.1</b>	<b>Apply critical thinking and problem-solving strategies during structured learning experiences.</b>

<b>MINI UNIT 3A</b>	
<b>Title:</b>	<b>Solving Trigonometric Equations</b>
<b>Duration:</b>	<b>3 weeks</b>
<b>Overview:</b>	<b>This unit focuses on trigonometric equations and their graphs. Students will solve trigonometric equations both algebraically and graphically. The inverses of trigonometric functions will be explored. Students will learn how to restrict the domain of the trigonometric functions in order to discover and graph its inverse.</b>
<b>Essential Outcomes - Upon completion of this course students will know (declarative):</b>	<b>Alignment to Standards</b>
<b>The relationship between solutions and x-intercepts of trigonometric graphs</b>	<b>F.TF.6 F.TF.7</b>

<b>The meaning of a complete solution in terms of trigonometry</b>	<b>F.TF.6 F.TF.7</b>
<b>The domain and range of inverse trigonometric functions</b>	<b>F.TF.7</b>
<b>Inverse trigonometric notation</b>	<b>F.TF.7</b>
<b>A variety of techniques to solve trigonometric equations</b>	<b>F.TF.7</b>
<b>Essential Outcomes - Upon completion of this course students will be able to (procedural):</b>	<b>Alignment to Standards</b>
<b>Solve trigonometric equations graphically</b>	<b>F.TF.6 F.TF.7</b>
<b>State the complete solution of a trigonometric equation</b>	<b>F.TF.6 F.TF.7</b>
<b>Define the domain and range of inverse trigonometric functions</b>	<b>F.TF.7</b>
<b>Use inverse trigonometric functions</b>	<b>F.TF.7</b>
<b>Work with a variety of techniques to solve trigonometric equations</b>	<b>F.TF.7</b>
<b>Essential Outcomes - Upon completion of this course students will understand (conceptual):</b>	<b>Alignment to Standards</b>
<b>The relationship between algebraic and graphical solutions to trigonometric functions</b>	<b>F.TF.6 F.TF.7</b>

<b>Why there is often more than one solution to trigonometric equations</b>	<b>F.TF.6 F.TF.7</b>
<b>The relationship between a trigonometric function and its inverse</b>	<b>F.TF.7</b>
<b>When and how to use various techniques to solve trigonometric equations</b>	<b>F.TF.7</b>
<b>Resources Mini Unit 3A:</b>	TI-84 Calculator, Textbook, Teaching Prompts, Interactive Whiteboard

<b>MINI UNIT 3B</b>	
<b>Title:</b>	<b>Trigonometric Identities</b>
<b>Duration:</b>	<b>5 weeks</b>
<b>Overview:</b>	<b>This unit focuses on trigonometric identities. Students will learn several strategies on how to use the identities, as well as prove them to be true.</b>
<b>Essential Outcomes - Upon completion of this course students will know (declarative):</b>	<b>Alignment to Standards</b>
<b>Trigonometric identities using graphs</b>	<b>F.TF.9</b>
<b>Strategies used to prove trigonometric identities</b>	<b>F.TF.9</b>
<b>The addition and subtraction identities for sine, cosine, and tangent</b>	<b>F.TF.9</b>
<b>The cofunction identities of trigonometry</b>	<b>F.TF.9</b>
<b>Various strategies to solve trigonometric identities</b>	<b>F.TF.9</b>

<b>Essential Outcomes - Upon completion of this course students will be able to (procedural):</b>	<b>Alignment to Standards</b>
Identify possible identities using graphs	F.TF.9
Apply strategies to prove identities	F.TF.9
Use the addition and subtraction identities for sine, cosine, and tangent functions	F.TF.9
Identify and graph the cofunction identities	F.TF.9
Use the appropriate identity rule to solve trigonometric identities	F.TF.9
<b>Essential Outcomes - Upon completion of this course students will understand (conceptual):</b>	<b>Alignment to Standards</b>
The difference between identities on the xy plane	F.TF.9
The appropriate strategy to use when proving an identity	F.TF.9
The effect of the addition and subtraction identities for sine, cosine, and tangent	F.TF.9
The graphs of the cofunction identities	F.TF.9
When and how to use the appropriate strategy to solve a trigonometric identity	F.TF.9
<b>Resources Mini Unit 3B:</b>	TI-84 Calculator, Textbook, Teaching Prompts, Interactive Whiteboard



## UMBRELLA UNIT 4

<b>Title:</b>	Trigonometric Applications, Analytic Geometry, and Systems of Equations
<b>Duration:</b>	8 weeks
<b>Essential Questions:</b>	<ul style="list-style-type: none"><li>- When is it necessary to use the Law of Sines to solve a triangle?</li><li>- How can you use the Binomial Theorem to expand binomials?</li><li>- How is a complex number converted to polar form?</li><li>- What is the difference between vectors and rays?</li><li>- How do you find the dot product for two vectors?</li><li>- How does the concept of distance relate to the concepts of ellipses and hyperbolas?</li><li>- How do you determine the shape of a translated conic section with graphing?</li><li>- How is the procedure of parameterization of conic sections used to solve real world problems?</li><li>- When would you use matrices in a real life application?</li><li>- How do you determine whether two matrices can be added, subtracted, multiplied, or solved?</li><li>- How do you solve a nonlinear system graphically?</li></ul>
<b>Summative Assessments:</b> (Assessment at the end the learning period)	Benchmark Post Test covering all concepts covered in Unit 4
<b>Formative Assessments:</b> (Ongoing assessments during the learning period)	<b>Homework:</b> to be given daily on each introduced topic <b>Quizzes:</b> <ul style="list-style-type: none"><li>- The Law of Sines and The Law of Cosines</li><li>- Finding Area of Triangles</li><li>- The Complex Plane and Polar Form for Complex Numbers</li><li>- Vectors</li><li>- Ellipses and Hyperbolas</li><li>- Parabolas</li></ul>

	<ul style="list-style-type: none"> <li>- Operations of Matrices</li> <li>- Solving Systems of Matrices</li> </ul> <b>Tests:</b> <ul style="list-style-type: none"> <li>- Trigonometric Applications</li> <li>- Analytic Geometry</li> <li>- Solving Systems of Equations</li> </ul>
<b>Differentiation:</b>	Variations can be made to daily homework assignments for students achieving above or below the majority. Self-discovery labs can be used in place or class discussion.
<b>CPI #</b>	<b>CUMULATIVE PROGRESS INDICATOR (CPI)</b>
<b>8.1.12.A.1</b>	<b>Construct a spreadsheet, enter data, and use mathematical or logical functions to manipulate data, generate charts and graphs, and interpret the results</b>
<b>21ST CENTURY LIFE AND CAREER (STANDARD 9)</b>	
<b>CPI #</b>	<b>CUMULATIVE PROGRESS INDICATOR (CPI)</b>
<b>9.1.12.A.1</b>	<b>Apply critical thinking and problem-solving strategies during structured learning experiences.</b>

<b>MINI UNIT 4A</b>	
<b>Title:</b>	<b>Trigonometric Applications</b>
<b>Duration:</b>	<b>3 weeks</b>
<b>Overview:</b>	<p><b>This unit focuses on the application of Trigonometry concepts. Students will learn how to solve triangles using both the Law of Sines and the Law of Cosines. A new formula for finding the area of a triangle using the Law of Sines is introduced. The unit continues into binomial expansion and the use of the Binomial Theorem. Students will be able to use complex numbers and graph them in the complex plane.</b></p>

Essential Outcomes - Upon completion of this course students will know (declarative):	Alignment to Standards
Oblique triangles and the Law of Sines and Cosines	G.SRT.10 G.SRT.11
The applications of the Law of Sines and Cosines to solve real-life problems	G.SRT.10 G.SRT.11
The area formula for triangles	G.SRT.9
The Binomial Theorem	A.APR.5
Binomial Expansion	A.APR.5
Complex numbers is the complex plane	N.CN.4 N.CN.5
Polar form of a complex number	N.CN.4 N.CN.5
Components and magnitude of vectors	N.VM.1 N.VM.2 N.VM.3 N.VM.4
Scalar multiplication, addition, and subtraction with vectors	N.VM.1 N.VM.2 N.VM.3 N.VM.4
Resultant forces in physical applications	N.VM.1 N.VM.2 N.VM.3 N.VM.4
Projection and component vectors	N.VM.1 N.VM.2

	<b>N.VM.3 N.VM.4</b>
<b>Essential Outcomes - Upon completion of this course students will be able to (procedural):</b>	<b>Alignment to Standards</b>
<b>Solve oblique triangles using the Law of Sines and Cosines</b>	<b>G.SRT.10 G.SRT.11</b>
<b>Solve real-life problems using the Law of Sines and Cosines</b>	<b>G.SRT.10 G.SRT.11</b>
<b>Use the area formula to find area of triangles</b>	<b>G.SRT.9</b>
<b>Expand the power of binomials using the Binomial Theorem</b>	<b>A.APR.5</b>
<b>Find the coefficient of a given term of a binomial expansion</b>	<b>A.APR.5</b>
<b>Graph a complex number in the complex plane</b>	<b>N.CN.4 N.CN.5</b>
<b>Find the absolute value of a complex number</b>	<b>N.CN.4 N.CN.5</b>
<b>Express a complex number in polar form</b>	<b>N.CN.4 N.CN.5</b>
<b>Perform polar multiplication and division</b>	<b>N.CN.4 N.CN.5</b>
<b>Calculate power and roots of complex numbers</b>	<b>N.CN.4 N.CN.5</b>
<b>Find and graph roots of unity</b>	<b>N.CN.4</b>

<b>Find the component and magnitude of a vector</b>	<b>N.VM.1 N.VM.2 N.VM.3 N.VM.4</b>
<b>Perform scalar multiplication of vectors, vector addition, and vector subtraction</b>	<b>N.VM.1 N.VM.2 N.VM.3 N.VM.4</b>
<b>Determine the direction angle of a vector</b>	<b>N.VM.1 N.VM.2 N.VM.3 N.VM.4</b>
<b>Determine resultant forces in physical applications</b>	<b>N.VM.1 N.VM.2 N.VM.3 N.VM.4</b>
<b>Determine projection and component vectors and apply them to physical applications</b>	<b>N.VM.1 N.VM.2 N.VM.3 N.VM.4</b>
<b>Essential Outcomes - Upon completion of this course students will understand (conceptual):</b>	<b>Alignment to Standards</b>
<b>The similarities and differences between the Law of Sines and the Law of Cosines</b>	<b>G.SRT.10 G.SRT.11</b>
<b>Which Law (Sine or Cosine) is appropriate given certain information</b>	<b>G.SRT.10 G.SRT.11</b>
<b>How the Law of Sines is used to find the area of a</b>	<b>G.SRT.9</b>

<b>triangle</b>	
<b>The use of the Binomial Theorem with regard to binomial expansion</b>	<b>A.APR.5</b>
<b>Properties of complex numbers and how they relate to real numbers</b>	<b>N.CN.4 N.CN.5</b>
<b>Power and roots of complex numbers</b>	<b>N.CN.4 N.CN.5</b>
<b>Roots of unity</b>	<b>N.CN.5</b>
<b>The relationship between vectors and matrices</b>	<b>N.VM.1 N.VM.2 N.VM.3 N.VM.4</b>
<b>The physical application of projection and component vectors</b>	<b>N.VM.1 N.VM.2 N.VM.3 N.VM.4</b>
<b>The physical application of resultant forces</b>	<b>N.VM.1 N.VM.2 N.VM.3 N.VM.4</b>
<b>Resources Mini Unit 4A:</b>	TI-84 Calculator, Textbook, Teaching Prompts, Interactive Whiteboard

<b>MINI UNIT 4B</b>	
<b>Title:</b>	<b>Analytic Geometry</b>
<b>Duration:</b>	<b>2 weeks</b>
<b>Overview:</b>	<b>This unit focuses on ellipses, hyperbolas, and translated conics. Students will learn the properties, equations, and graphs of these relations. Students will gain an understanding of the polar coordinate system and how to convert coordinates from the polar system to the rectangular system.</b>
<b>Essential Outcomes - Upon completion of this course students will know (declarative):</b>	<b>Alignment to Standards</b>
<b>Properties and graphs of ellipses and hyperbolas</b>	<b>G.CPE.3</b>
<b>Properties and graphs of parabolas</b>	<b>G.CPE.3</b>
<b>Properties and graph of translated conics</b>	<b>G.CPE.3</b>
<b>The polar coordinate system</b>	<b>N.CN.4</b>
<b>Graphing in the polar coordinate system</b>	<b>N.CN.4</b>
<b>Essential Outcomes - Upon completion of this course students will be able to (procedural):</b>	<b>Alignment to Standards</b>
<b>Define and write the equation of ellipses and hyperbolas</b>	<b>G.CPE.3</b>
<b>Identify important characteristics and graph ellipses and hyperbolas</b>	<b>G.CPE.3</b>
<b>Define and write the equation of a parabola</b>	<b>G.CPE.3</b>
<b>Identify important characteristics and graph</b>	<b>G.CPE.3</b>

parabolas	
Graph and write the equation of a translated conic	G.CPE.3
Determine the shape of a translated conic without graphing	G.CPE.3
Locate points in the polar coordinate system and convert points from polar to rectangular systems	N.CN.4
<b>Essential Outcomes - Upon completion of this course students will understand (conceptual):</b>	<b>Alignment to Standards</b>
The similarities and differences of ellipses and hyperbolas	G.CPE.3
The distinguishing characteristics of parabolas and translated conics	G.CPE.3
The similarities and differences between the rectangular system and the polar system	N.CN.4
<b>Resources Mini Unit 4B:</b>	TI-84 Calculator, Textbook, Teaching Prompts, Interactive Whiteboard

<b>MINI UNIT 4C</b>	
<b>Title:</b>	<b>Systems of Equations</b>
<b>Duration:</b>	<b>3 weeks</b>
<b>Overview:</b>	<b>This unit focuses on solving systems of equations. Students will learn several different methods for solving systems, including elimination and substitution, Students will learn how to perform</b>



	<b>operations on matrices and apply these operations to solve systems. The unit concludes with real-life applications of these concepts.</b>
<b>Essential Outcomes - Upon completion of this course students will know (declarative):</b>	<b>Alignment to Standards</b>
<b>Systems of equations and how to solve them</b>	<b>N.VM.6 A.REI.11</b>
<b>Properties of matrices and performing matrix multiplication, addition, and subtraction</b>	<b>N.VM.6 N.VM.7 N.VM.8 N.VM.9 N.VM.10</b>
<b>Matrix inverses</b>	<b>N.VM.6 N.VM.7 N.VM.8 N.VM.9 N.VM.10</b>
<b>How to apply matrix properties to solve systems of equations</b>	<b>N.VM.6 N.VM.7 N.VM.8 N.VM.9 N.VM.10</b>
<b>Essential Outcomes - Upon completion of this course students will be able to (procedural):</b>	<b>Alignment to Standards</b>
<b>Solve a system of equations using various methods</b>	<b>N.VM.6 A.REI.11</b>
<b>Add, subtract, and multiply matrices</b>	<b>N.VM.6 N.VM.7</b>

Find the inverse of a matrix when possible	N.VM.6 N.VM.7 N.VM.8 N.VM.9 N.VM.10
Solve square systems of equations using inverse matrices	N.VM.6 A.REI.11
Solve applications using systems	N.VM.6 N.VM.7 N.VM.8 N.VM.9 N.VM.10
<b>Essential Outcomes - Upon completion of this course students will understand (conceptual):</b>	<b>Alignment to Standards</b>
When to use the appropriate method to solve a system of equations	N.VM.6 A.REI.11
When it is possible to perform operations on matrices	N.VM.6 N.VM.7 N.VM.8 N.VM.9 N.VM.10
How matrices are used to solve systems of equations	N.VM.6 A.REI.11
What it means to be a consistent or inconsistent system	N.VM.6 N.VM.7 N.VM.8

		<b>N.VM.9</b> <b>N.VM.10</b>
<b>When systems can be used to solve real-life applications</b>		<b>N.VM.6</b> <b>N.VM.7</b> <b>N.VM.8</b> <b>N.VM.9</b> <b>N.VM.10</b>
<b>Resources Mini Unit 4C:</b>	TI-84 Calculator, Textbook, Teaching Prompts, Interactive Whiteboard	

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